Autovectores a partir de autovalores

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Antes de empezar...

Finding the Resistance Distance and Eigenvector Centrality from the Network's Eigenvalues

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There are different measures to classify a network's data set that, depending on the problem, have different success. For example, the resistance distance and eigenvector centrality measures have been successful in revealing ecological pathways and differentiating between biomedical images of patients with Alzheimer's disease, respectively. The resistance distance measures the effective distance between any two nodes of a network taking into account all possible shortest paths between them and the eigenvector centrality measures the relative importance of each node in the network. However, both measures require knowing the network's eigenvalues and eigenvectors – eigenvectors being the more computationally demanding task. Here, we show that we can closely approximate these two measures using only the eigenvalue spectra, where we illustrate this by experimenting on elemental resistor circuits and paradigmatic network models – random and small-world networks. Our results are supported by analytical derivations, showing that the eigenvector centrality can be perfectly matched in all cases whilst the resistance distance can be closely approximated. Our underlying approach is based on the work by Denton, Parke, Tao, and Zhang arXiv:1908.03795 (2019)], which is unrestricted to these topological measures and can be applied to most problems requiring the calculation of eigenvectors.

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El punto de partida

EIGENVECTORS FROM EIGENVALUES

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ABSTRACT. We present a new method of succinctly determining eigenvectors from eigenvalues. Specifically, we relate the norm squared of the elements of eigenvectors to the eigenvalues and the submatrix eigenvalues.

EIGENVECTORS FROM EIGENVALUES: A SURVEY OF A BASIC IDENTITY IN LINEAR ALGEBRA

PETER B. DENTON, STEPHEN J. PARKE, TERENCE TAO, AND XINING ZHANG

ABSTRACT. If A is an $n \times n$ Hermitian matrix with eigenvalues $\lambda_1(A), \dots, \lambda_n(A)$ and $i, j = 1, \dots, n$, then the j^{th} component $v_{i,j}$ of a unit eigenvector v_i associated to the eigenvalue $\lambda_i(A)$ is related to the eigenvalues $\lambda_1(M_j), \dots, \lambda_{n-1}(M_j)$ of the minor M_j of A formed by removing the j^{th} row and column by the formula

$$|v_{i,j}|^2 \prod_{k=1; k \neq i}^{n} (\lambda_i(A) - \lambda_k(A)) = \prod_{k=1}^{n-1} (\lambda_i(A) - \lambda_k(M_j)) .$$

We refer to this identity as the *eigenvector-eigenvalue identity*. Despite the simple nature of this identity and the extremely mature state of development of linear algebra, this identity was not widely known until very recently. In this

Eigenvalues: the Rosetta Stone for Neutrino Oscillations in Matter

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We present a new method of exactly calculating neutrino oscillation probabilities in matter. We leverage the "eigenvector-eigenvalue identity" to show that, given the eigenvalues, all mixing angles in matter follow surprisingly simply. The CP violating phase in matter can then be determined from the Toshev identity. Then, to avoid the cumbersome expressions for the exact eigenvalues, we have applied previously derived perturbative, approximate eigenvalues to this scheme and discovered them to be even more precise than previously realized. We also find that these eigenvalues converge at a rate of five orders of magnitude per perturbative order which is the square of the previously realized expectation. Finally, we provide an updated speed versus accuracy plot for oscillation probabilities in matter, to include the methods of this paper.

Identidad autovector-autovalor

Para una matriz hermítica $\mathbf{A}$ con autovalores no degenerados:

$$\left|[\vec{\psi}_n]_i\right|^2 = \frac{\prod_{k=1}^{N-1}[\lambda_n(\mathbf{A}) - \lambda_k(\mathbf{M}_i)]}{\prod_{k=1;\,k\neq n}^{N}[\lambda_n(\mathbf{A}) - \lambda_k(\mathbf{A})]},$$

donde $\mathbf{M}_i$ es la submatriz obtenida a partir de $\mathbf{A}$, removiendo la fila y columna i-ésima.

Métricas de red

- **Distancia resistiva:**

$$\rho(i,j) = \sum_{n=2}^{N} \frac{1}{\lambda_n(\mathbf{L})} \left| [\vec{\phi}_n]_i - [\vec{\phi}_n]_j \right|^2$$

Con $\mathbf{L} = \mathbf{D} - \mathbf{A}$ y $\mathbf{L}\vec{\phi}_n = \lambda_n(\mathbf{L})\vec{\phi}_n$

- **Autovector central:** autovector de la matriz de adyacencia (A) asociado al mayor autovalor. Todos sus componentes son positivos.

Métricas de red

- **Distancia Resistiva:**

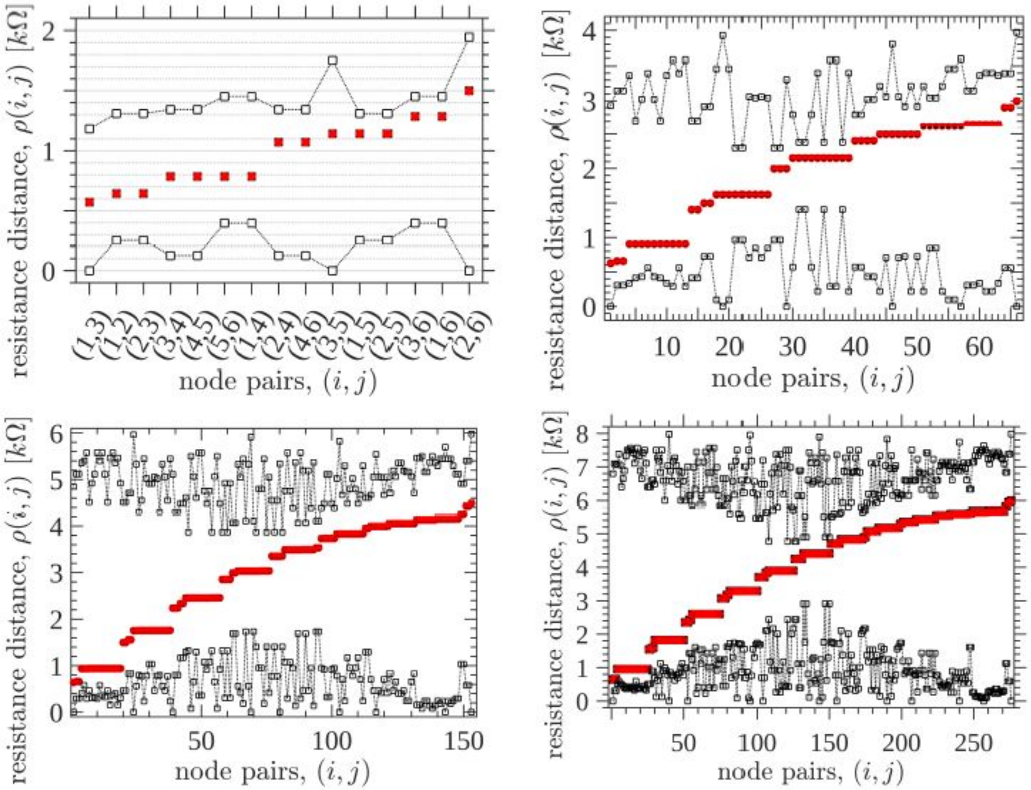
$$\left|[\vec{\phi}_n]_i - [\vec{\phi}_n]_j\right|^2 = \left|[\vec{\phi}_n]_i\right|^2 + \left|[\vec{\phi}_n]_j\right|^2 - 2[\vec{\phi}_n]_i[\vec{\phi}_n]_j^*$$

$$\rho_{up}(i,j) = \sum_{n=2}^{N} \frac{1}{\lambda_n(\mathbf{L})} \left(\left|[\vec{\phi}_n]_i\right|^2 + \left|[\vec{\phi}_n]_j\right|^2 + 2\left|[\vec{\phi}_n]_i\right| \left|[\vec{\phi}_n]_j\right| \right)$$

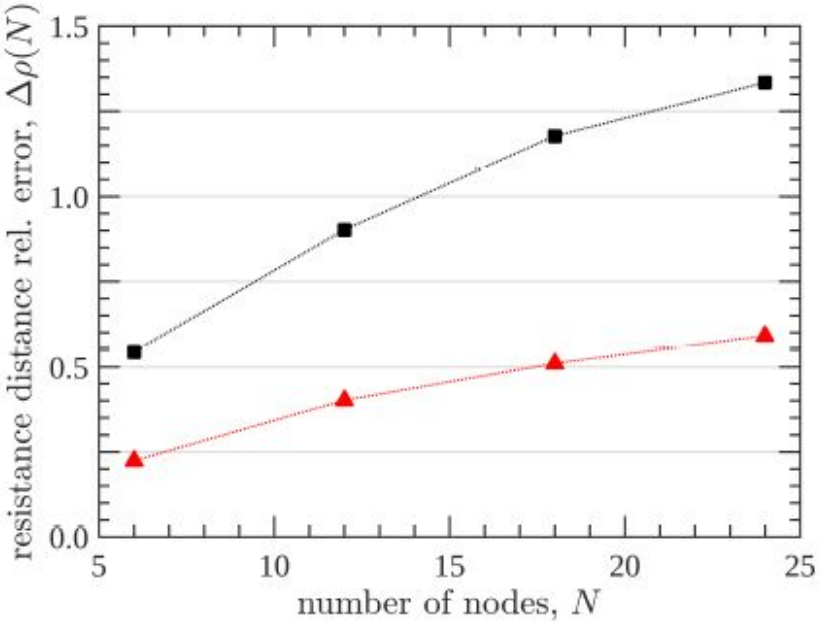
$$\rho_{down}(i,j) = \sum_{n=2}^{N} \frac{1}{\lambda_n(\mathbf{L})} \left(\left|[\vec{\phi}_n]_i\right|^2 + \left|[\vec{\phi}_n]_j\right|^2 - 2\left|[\vec{\phi}_n]_i\right| \left|[\vec{\phi}_n]_j\right| \right)$$

$$\rho_{approx}(i,j) = \frac{1}{2}\left[\rho_{up}(i,j) + \rho_{down}(i,j)\right]$$

Resultados Experimentales

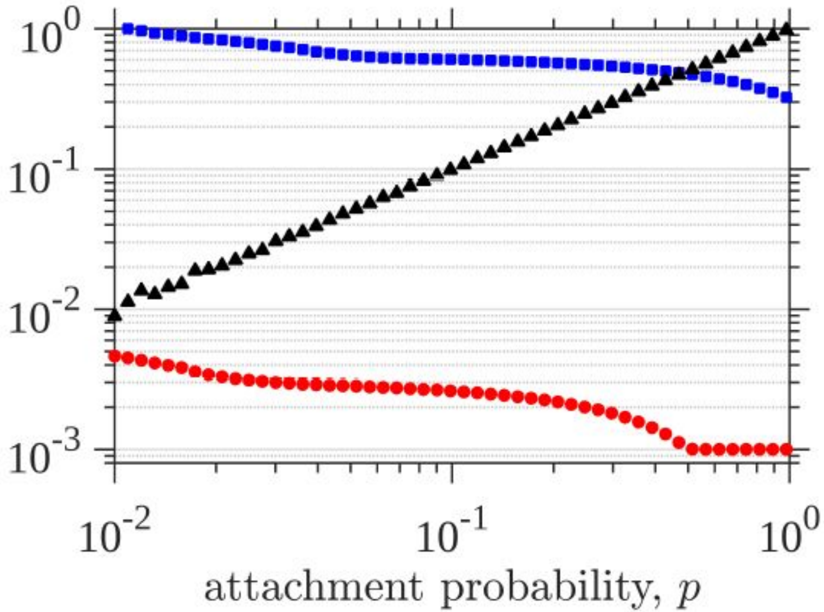


Resultados Experimentales

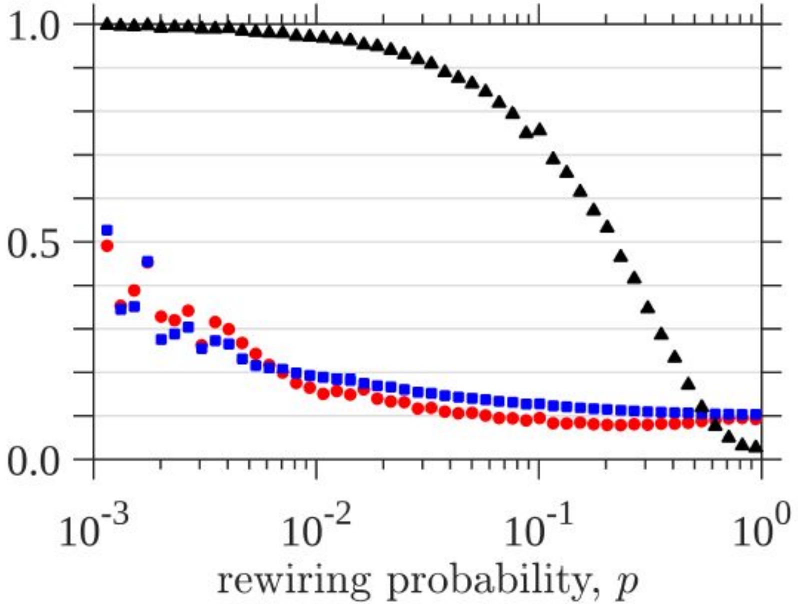


$$\Delta\rho(N) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j>i}^{N} \left| 1 - \frac{\rho_\alpha(i,j)}{\rho_e(i,j)} \right|$$

Resultados numéricos



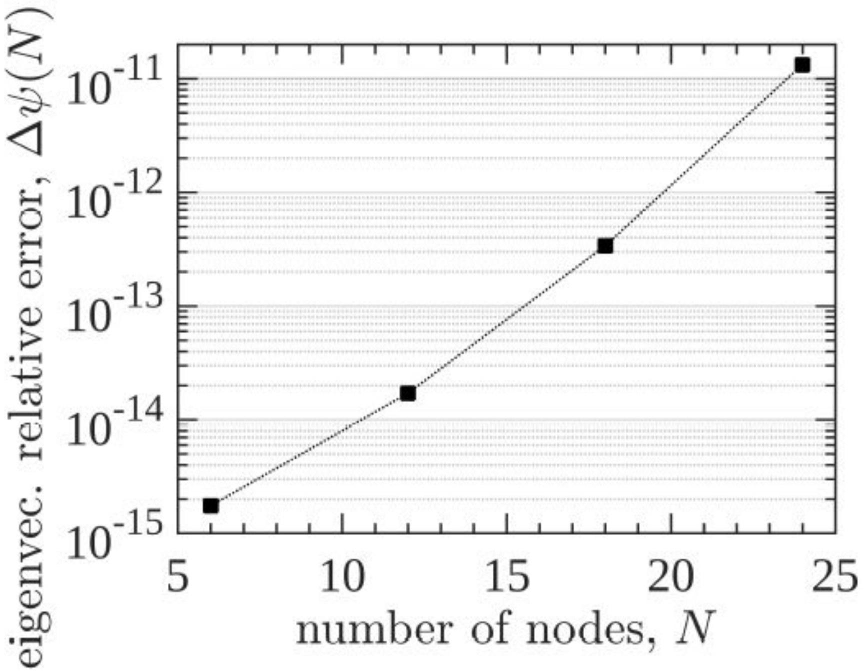
Erdos-Renyi

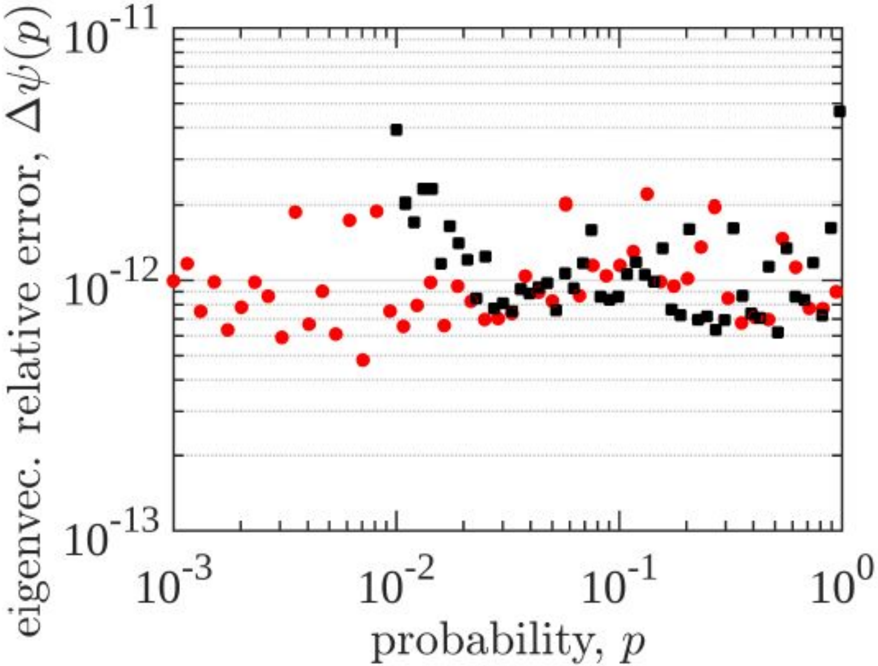


Watts-Strogatz

N=1000

Autovector Central





Conclusiones

- El autovector central se calcula exactamente (solo errores numéricos)
- La aproximación de distancia resistiva es mejor para redes aleatorias.
- El uso de la identidad no se limita a las medidas presentadas.
- Generalización de distancia resistiva: ¿es un método eficiente?