Approximating generating Markov partitions (GMP): from time series to symbolic sequences

Seminarios del grupo Física No Lineal, 2020

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N. Rubido, C. Grebogi, and M. S. Baptista, "Entropy-based generating Markov partitions for complex systems", *Chaos* **28**, 033611 (2018).

Contexto: Ergodicidad
Generating Markov Partitions for Time-series Analysis

- **Ergodicidad:**

  Un proceso es ergodico si los **promedios en ensembles** (espaciales) son iguales a los **promedios en el tiempo** con probabilidad 1.

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} X_t(\omega_i) = \frac{1}{T}\sum_{t=1}^{T} X_t(\omega) \to \mu$$

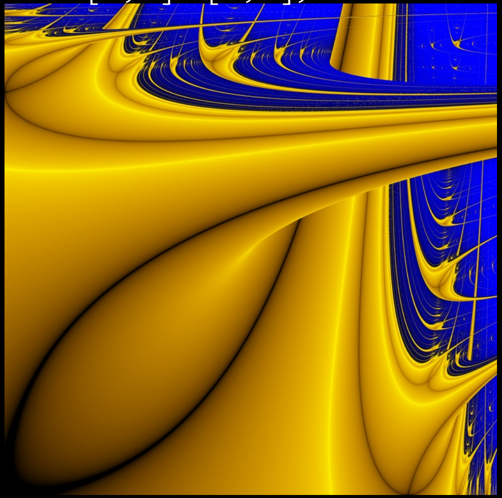
  Por ejemplo, los exponentes de Lyapunov:

$$\lambda = \int_{\Omega} \ln\left(DF(x)\right)\rho(x)\, d^D x = \lim_{T\to\infty} \ln\left(\left|DF(x)^{1/T}\right|\right)$$

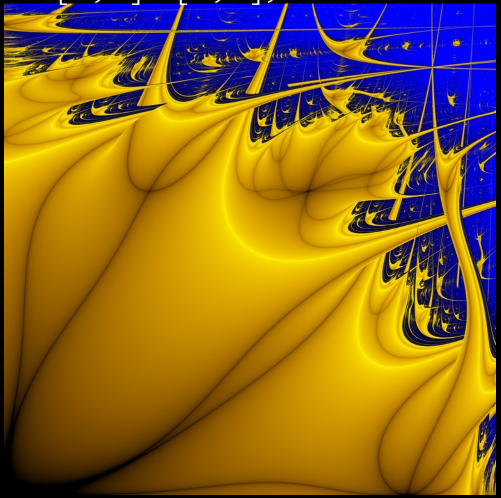
- **Mapa logístico fractal** (extensión de Markus):

$$\lambda = \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} \log\left|\frac{dx_{n+1}}{dx_n}\right| = \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} \log\left|r_n(1-2x_n)\right|$$

$r \in [2,4]\times[2,4], secs. AB$



$r \in [2,4]\times[2,4], secs. AABAB$



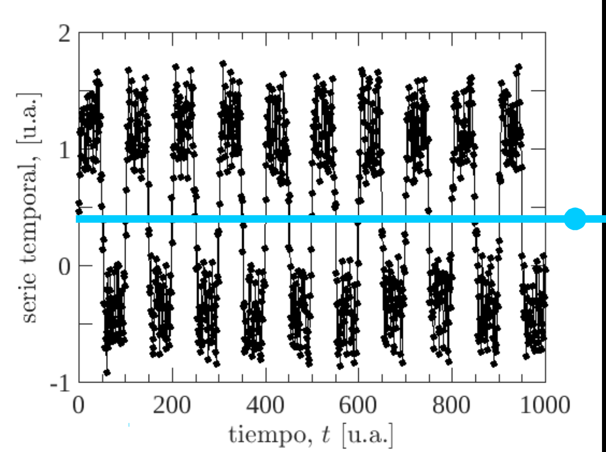
Contexto: Distribuciones de Probabilidad
Generating Markov Partitions for Time-series Analysis

Señales (tiempo)
$\{x(t_i)\}_{i=1}^{T} \in \mathbb{R}$

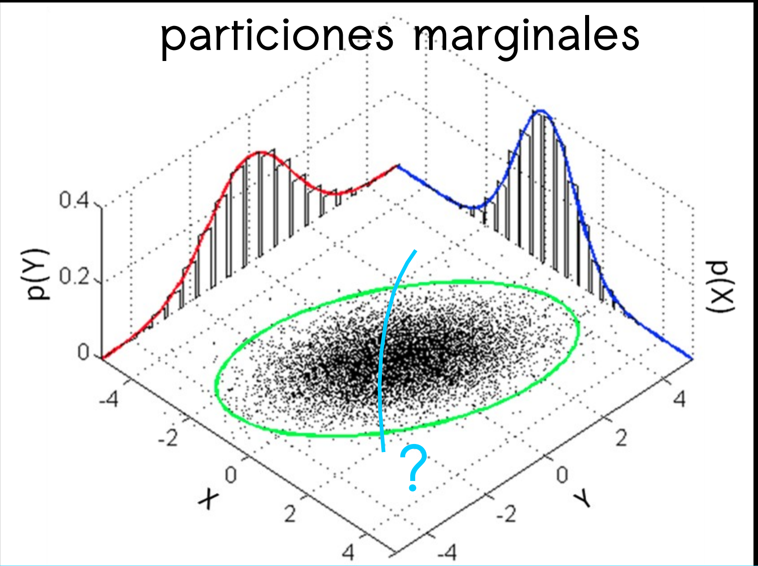
Partición
(~ def. bines)

Serie simbólica
$\{s(i)\}_{i=1}^{N_s} \in \mathbb{N}$

(**+ simple**, pero con
= información)



+ resolución (+ bines) →
medida invariante Prob.



¿∃ **alguna partición** que conserve las **propiedades invariantes** del flujo y la **dist. de probabilidad**?

Problema: encontrar particiones generatrices

Generating Markov Partitions for Time-series Analysis

Señales (tiempo)
$\{x(t_i)\}_{i=1}^{T} \in \mathbb{R}$

Partición generatriz de Markov

Serie simbólica
$\{s(i)\}_{i=1}^{N_s} \in \mathbb{N}$

$$x_{n+1} = 1 + u(x_n \cos t_n - y_n \sin t_n), \quad t_n = 0.4 - \frac{6}{1 + x_n^2 + y_n^2}.$$
$$y_{n+1} = u(x_n \sin t_n + y_n \cos t_n),$$

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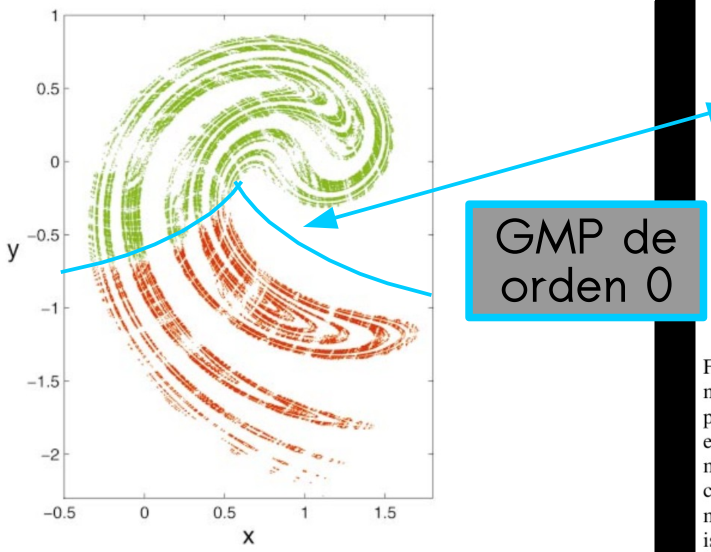

FIG. 2. (Color.) Orbit points up to period-18 for the Ikeda-Hammel-Jones-Moloney attractor in Eq. (2) colored according to their symbolic representation: green and red dots represent orbit points encoded with symbols **0** and **1**, respectively.

[4] (Davidchack, PRE 2000)

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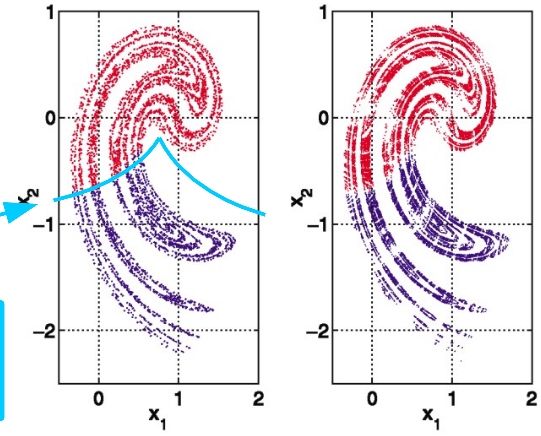


FIG. 1 (color online). Left: partition estimated by J_{sfnn} optimization on 2000 data points from the Ikeda map. Right: partition calculated with foreknowledge of UPOs, numerically extracted from the equation of motion. The partition we estimate from observed data alone is quite close to a presumably correct partition, calculated from the method of [6]. The measure on the two figures is not the same: the left-hand figure is a sample of the natural measure, whereas the right-hand figures shows UPOs up to period 16. They avoid regions of homoclinic tangencies, contributing to the blank spaces.

[6] (Kennel, PRL 2003)

Ejemplo: solución analítica para mapa logístico

Generating Markov Partitions for Time-series Analysis

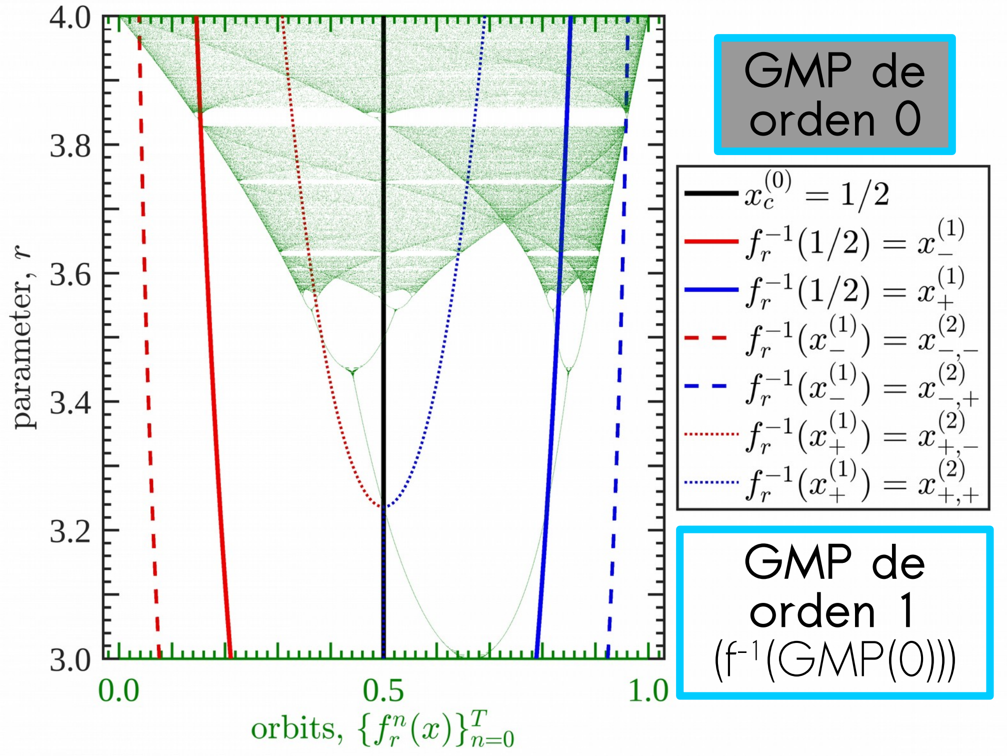
Señales (tiempo)

$\{x(t_i)\}_{i=1}^{T} \in \mathbb{R}$

Partición generatriz de Markov

Serie simbólica

$\{s(i)\}_{i=1}^{N_s} \in \mathbb{N}$



GMP de orden 0

GMP de orden 1 (f^{-1}(GMP(0)))

$f_r(x) = r\,x(1-x)$

Órbita por x_0 (**señal**):

$x_t = f_r(x_{t-1}) = \ldots = f_r^t(x_0)$

$\Rightarrow \{x_t\}_{t=0}^{T} = \{f_r^t(x_0)\}_{t=0}^{T}$

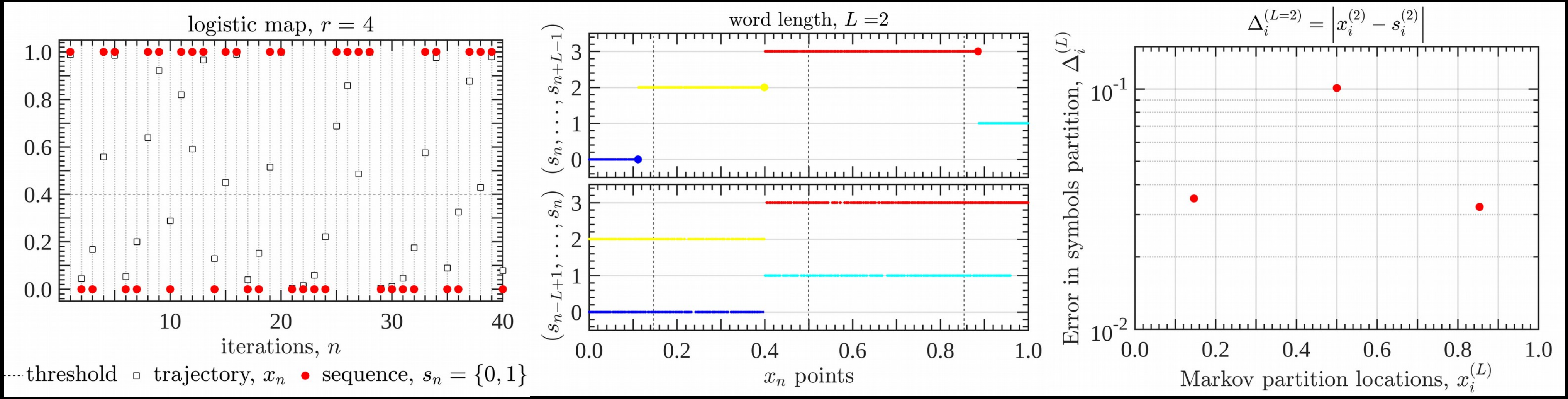
Exponente Lyapunov
(divergencia temporal):

$$\lambda_r = \lim_{n\to\infty} \sum_{t=0}^{T} \frac{\ln(|f'(x_t)|)}{T}$$

$$\lambda_{r=4} = \ln(2)$$

Ejemplo: aproximando GMPs del mapa logístico

Generating Markov Partitions for Time-series Analysis



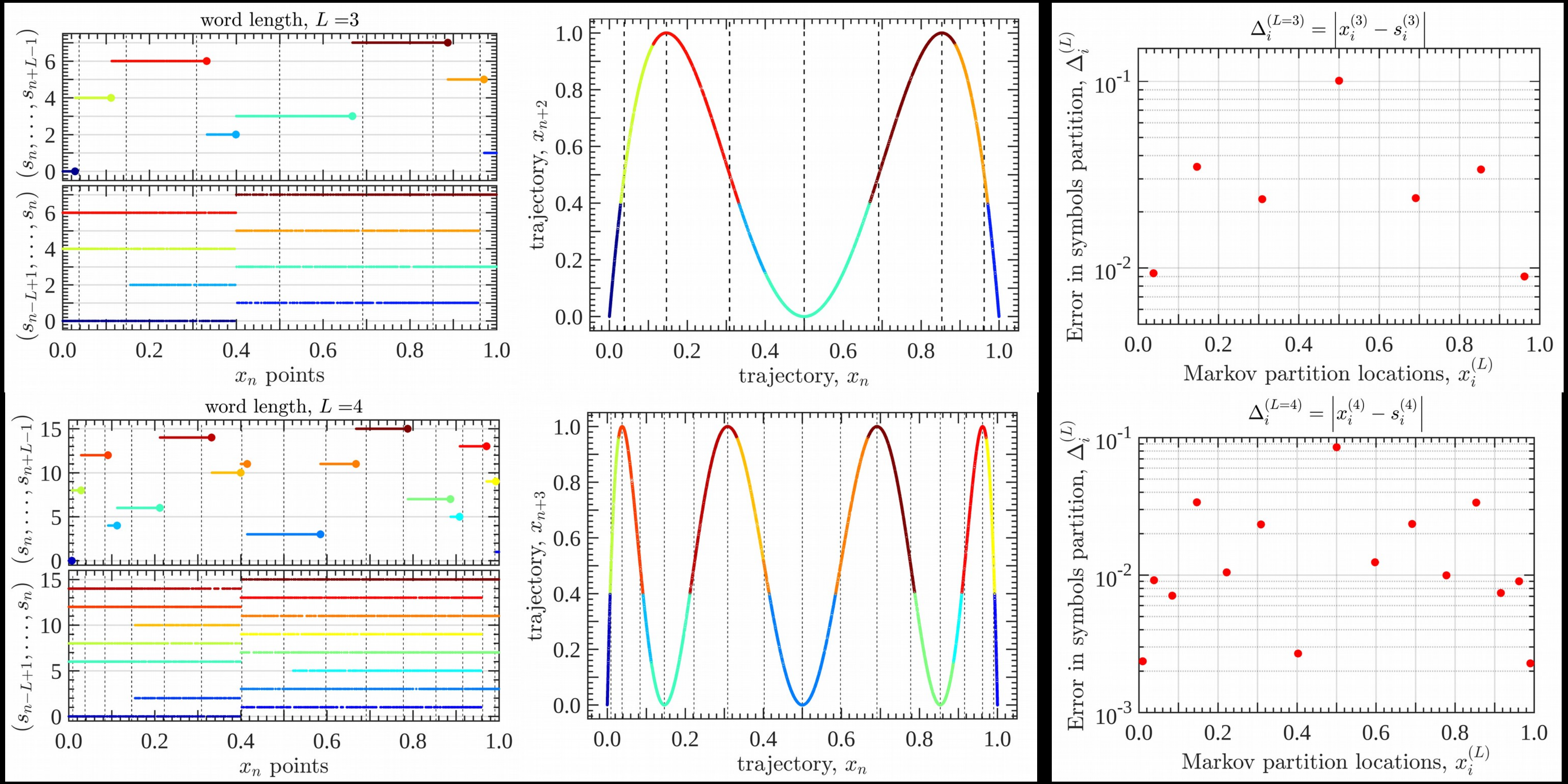
Señales (tiempo)

$\{x(t_i)\}_{i=1}^{T=512} \in \mathbb{R}$

Serie simbólica

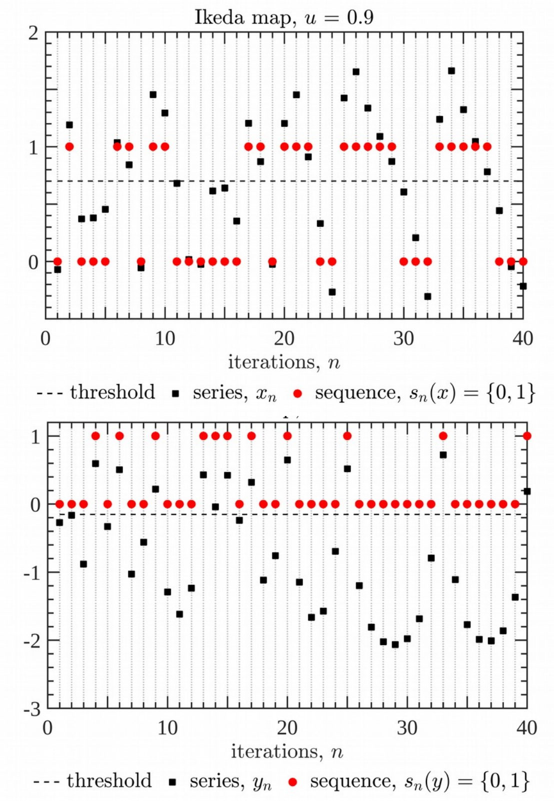
$\{s(i)\}_{i=1}^{N_s=T-L+1} \in \mathbb{N}$

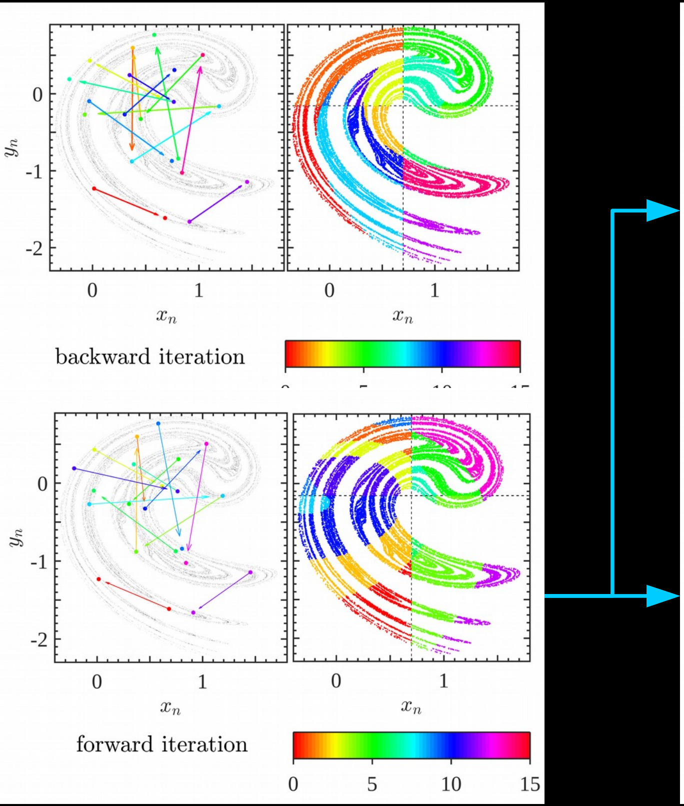
Distancias a las GMPs (orden 1 y 2)

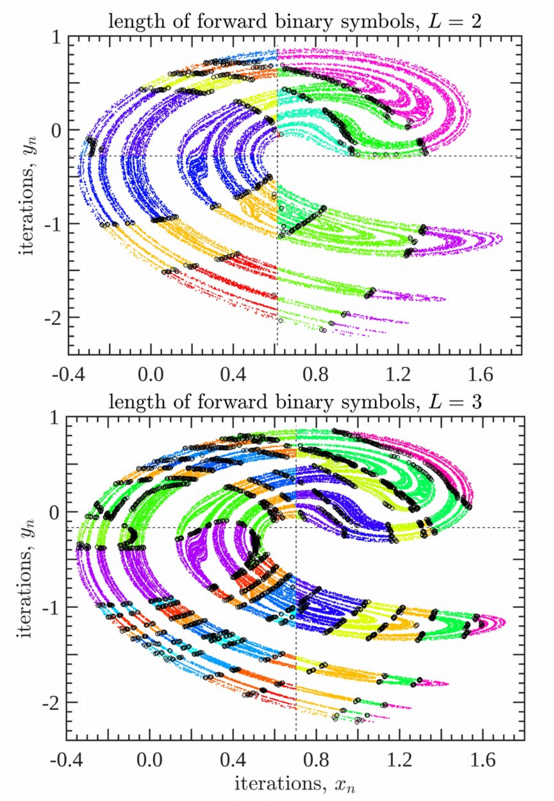

word length, L =3
$(s_n, \ldots, s_{n+L-1})$
$(s_{n-L+1}, \ldots, s_n)$
x_n points
trajectory, x_{n+2}
trajectory, x_n
$\Delta_i^{(L=3)} = \left| x_i^{(3)} - s_i^{(3)} \right|$
Error in symbols partition, $\Delta_i^{(L)}$
Markov partition locations, $x_i^{(L)}$
word length, L =4
$(s_n, \ldots, s_{n+L-1})$
$(s_{n-L+1}, \ldots, s_n)$
x_n points
trajectory, x_{n+3}
trajectory, x_n
$\Delta_i^{(L=4)} = \left| x_i^{(4)} - s_i^{(4)} \right|$
Error in symbols partition, $\Delta_i^{(L)}$
Markov partition locations, $x_i^{(L)}$

Ejemplo: aproximando GMPs del mapa de Ikeda

Generating Markov Partitions for Time-series Analysis

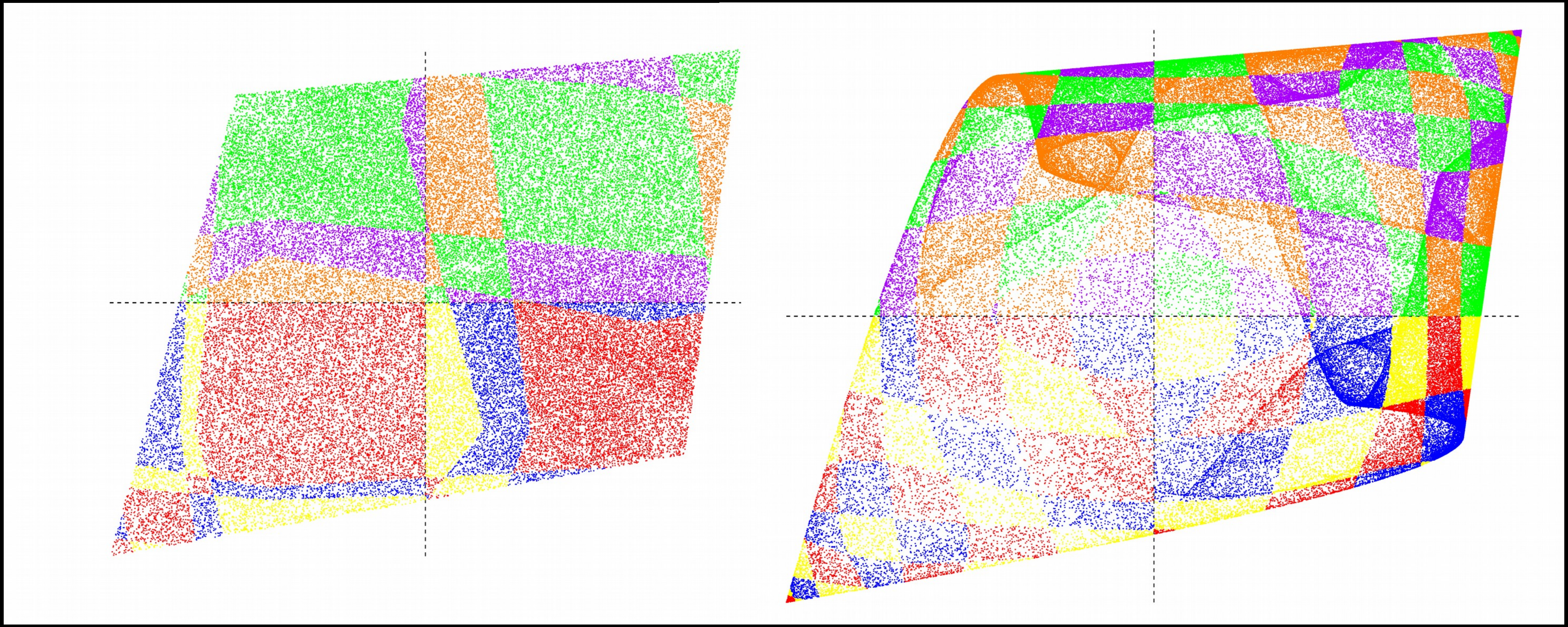






Ejemplo: mapas acoplados (tienda y logísticos)

Generating Markov Partitions for Time-series Analysis



References

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(6) M. B. Kennel and M. Buhl, "Estimating good discrete partitions from observed data: Symbolic false nearest neighbors", *Physical Review Letters* **91**, 084102 (2003).

(7) H. Teramoto and T. Komatsuzaki, "How does a choice of Markov partition affect the resultant symbolic dynamics?", *Chaos* **20**, 037113 (2010).